Blowing Hard Is Not All We Want: Quantity vs Quality of Wind Power in the Smart Grid

Fanxin Kong† Chuansheng Dong‡ Xue Liu† Haibo Zeng†
† School of Computer Science, McGill University, Canada
‡ Department of Electrical & Computer Engineering, McGill University, Canada

{fanxin.kong, chuansheng.dong}@mail.mcgill.ca, xueliu@cs.mcgill.ca, haibo.zeng@mcgill.ca

Abstract—The growing awareness about global climate change has boosted the need to mitigate greenhouse gas emissions from existing power systems and spurred efforts to accelerate the integration of renewable energy sources (e.g., wind and solar power) into the electrical grid. A fundamental difficulty here is that renewable energy sources are usually of high variability. The electrical grid must absorb this variability through employing many additional operations (e.g., operating reserves, energy storage), which will largely raise the cost of electricity from renewable energy sources. To make it affordable, numerous advancements in technologies and methods for the smart grid are required. In this paper, we will confine ourselves to one of them: how to plan the construction of wind farms with high capacity and low variability locally and distributedly. We first study the characteristics of both wind resources and wind turbines and present a more accurate wind power evaluation method based on Gaussian Regression. Then, we analyze a trade-off between wind power’s quantity and quality and propose an approach to optimally combine different types of wind turbines to balance the trade-off for a specific site. Finally, we explore geographical diversity among different sites and develop an extended approach that jointly optimizes the combination of sites and turbine types. Extensive experiments using the realistic historical wind resource data are conducted for either of the local and distributed case. Encouraging results are shown for the proposed approaches and some interesting insights are also provided.

I. INTRODUCTION

Existing power systems create heavy environmental impacts and are a prime cause for the current climate change. For example, in 2011, total U.S. energy-related carbon dioxide (CO2) emissions by the electric power sector were 2,166 million metric tons and contribute about 40% of total U.S. energy-related CO2 emissions [1]. In recent years, concerns about climate change have grown of the need to reduce these emissions from this sector and stimulated efforts to speed up the integration of energy from renewable sources (e.g., wind and solar power) into electricity supply industries. On the other hand, there is a burning need for upgrading our current electrical grid into a smart grid, which is a robust, sustainable power transmission and distribution system that is intelligent, reliable and environment friendly. The smart grid is the key enabler for deep integration of renewable energy sources into our power systems [2], [3]. President Barack Obama advocated renewable energy as well as the smart grid and announced several billion dollars in U.S. government support along with private investment towards this end.

The biggest obstacle here is that renewable energy sources are highly variable. They are fundamentally different from conventional fossil fuel power generation (such as coal and natural gas) which is dispatchable and easily controlled. The variability of renewable energy is characterized by three distinct properties: (i) non-dispatchability or cannot be controlled on demand; (ii) intermittency or highly fluctuates; (iii) uncertainty or cannot be known in advance. The variability has little effect on the electrical grid if renewable power generation is kept at low penetration levels.

However, as the penetration goes deeper (e.g. 30% of total electricity demand), it needs much more efforts including power curtailment, energy storage technologies and operating reserves to smooth out the variability. The more variability and the deeper penetration renewable energy has, the higher the above solutions will cost. For example, the electrical grid would need large quantities of operating reserves (or conventional back-up power) and/or non-existent large-scale, centralized energy storage to tolerate the aggregated variability. To lower the cost of the electricity generated from renewable energy sources, it requires various advancements in technologies and methods drawn from optimization, modeling, and control for the smart grid, such as renewable energy forecasting [4]–[7], joint optimization of renewable energy and storage [8], matching variable generation with adjustable demand [9], [10], construction planning for renewable power plants [11], [12], and so on. In this paper, we study one of these issues: how to plan the construction of renewable power plants with both high capacity and low variability locally and distributedly.

We focus ourselves on the construction planning issue for wind farms. A wind farm usually consists of tens to hundreds of individual wind turbines. They convert wind energy into electricity and are connected to the electrical power transmission network. Renewable power plants’ construction planning is an extremely complicated decision making problem and involves a process of balancing diverse technical, economic, social and ecological aspects. The main concern of this paper is to address one fundamental issue: the quantity and quality of renewable energy sources, which is related to either of the above four aspects. This paper can be seen as a significant complement to the exiting construction planning work such as [11]–[13]. One of the conventional cost reduction approaches is to sit power plants in the locations with the most abundant
renewable resource such as where wind blows hardest or where sun irradiates strongest. This quantity-oriented approach was rather effective in the old ages when the power generation from conventional energy sources was dominant. However, as the era of deep integration of renewable energy sources comes with the smart grid, the cost of electricity generation is also heavily dependent on the quality or the magnitude of variability of renewable energy sources. Therefore, a thoughtful cost reduction approach should take account of both the quantity and quality of renewable energy sources at the same time when planning the construction of renewable power plants. However, there is always a trade-off between the two objectives, i.e., a site with large quantity usually has high variability and vice versa, due to the nature of wind resource. We propose novel solutions to deal with this issue. In specific, we make three contributions as follows.

First, we analyze the characteristics of power conversion for each type of wind turbine and then propose a more accurate wind power evaluation method based on Gaussian Regression. There are two motivations here. One is that, some existing works that address the wind farms’ construction planning issue employ wind speed or power density as a proxy to evaluate the quantity and/or quality for a site [11], [12], [14], [15]. The power density is simply calculated by cubing wind speed. These works can yield some misleading results on wind power assessment, since they totally overlook how much the wind resource could be converted into usable power or electricity by wind turbines and how well the output power is. The inaccurate evaluation will cause the cost rise if with a deep penetration level of renewable energy. The other motivation comes from one crucial observation that different types of wind turbines have different power characteristics [16]–[21]. That is, different turbine types have different quantities (or capacity factors) and qualities at power generation for a site. There is one optimal type of wind turbine for each site.

Second, we give some key observations through analyzing the evaluation result for each individual turbine type using the Gaussian Regression based method. Based on them, we propose an optimal wind turbine combination approach to balance the trade-off between wind power’s quantity and quality for a local site.

Finally, we explore the geographical diversity among different sites and address further how to balance the trade-off through jointly optimizing the combination of different sites and types of wind turbines. For a given requirement on the quantity of wind power generation, the approach optimizes the quality; while for a given requirement on quality, the approach maximizes the quantity. We conduct extensive experiments based on a large amount of real-world meteorological data trace [22], and show encouraging results of the proposed approaches. We also provide some interesting insights based on result analysis.

The rest of the paper is organized as follows. Section II introduces an overview of wind resource and provides the Gaussian Regression based evaluation method. Section III presents the optimal wind turbine combination method. Section IV explores geographical diversity and gives an extended approach to jointly optimize the combination of sites as well as turbine types. Section V concludes this paper and points out the future work.

II. WIND POWER QUANTITY AND QUALITY EVALUATION

The most important step when determining the locations for constructing wind farms is the wind power’s quantity and quality evaluation. The former criterion is related to the estimated average annual capacity of the usable power potentially generated by a site, while the latter one refers to the estimated variance of that. Some existing evaluation methods employed wind speed as a proxy, which gives a rough description of the power capacity. A method with higher evaluation accuracy is needed as the percentage of the wind power integrated into the electrical grid grows. In this section, we analyze the characteristics of different types of wind turbines based on data from real-world product sheet and propose a method to achieve accurate evaluation.

A. Understanding Wind Resource

Wind speed data are usually measured by observing equipments with a certain height (e.g. ~ 10 meters), which is in most cases much smaller than the height of wind turbines (at least tens of meters). The 1/7th wind profile power law [23] is used to assess the wind speed at the height of a specific turbine (≥~ 50 meters):

$$\left(\frac{v}{v_r}\right) = \left(\frac{h}{h_r}\right)^{\alpha},$$

(1)

where $v$ is the wind speed at height $h$; $v_r$ is the known wind speed at a reference height $h_r$; the friction coefficient $\alpha = \frac{1}{7}$ or 0.143. Though $\alpha$ can vary from 0.1 (calm water) to 0.4 (large city with tall buildings) [24], 0.143 is the most common value suitable for most observation sites. The wind resource’s assessment of a site can be achieved by calculating the average value and/or variance of the historical wind speed adjusted by Equation (1).

One alternative approach to evaluate wind resource is to adopt the wind power density $P$ (W/m$^2$), which has a cubic relation with the wind speed $(v)$ [25]:

$$P = \frac{1}{2} C_p \rho v^3,$$

(2)

where $\rho$ is the air density, and $C_p$ is the power coefficient denoting how much power that can be extracted from the wind, which is limited by Betz’ Law and cannot be greater than 59.3%. The power coefficient is usually set constant when assessing wind resource in the literature such as [11], [12], [26]–[29].

B. Understanding Wind Turbines’ Characteristics

Both of the two approaches above solely focus on one side of the coin: the quantity and quality of the wind resource that a site possesses, while overlook the other side of the coin: how much the wind resource could be converted into usable power by wind turbines and how well the output power would
be. Due to the technological limitations of modern turbines, the power that can be extracted from the wind resource is only part of the total it possesses. The proportion, a.k.a, the turbine efficiency or power coefficient, varies a lot with the wind speed [16]–[21]. The maximum efficiency can reach at 50% while the minimum is close to 0%. For example, from Figure 1, we can see that as the wind speed grows, the power coefficient (the red curve) first increases and then starts to decrease from a turning point (10 m/s). Some misleading results would come out if using the Equation (2) with one constant coefficient. Besides this characteristic, there are at least other three that we need to take account of, in order to make a more accurate evaluation of the wind power that a site can potentially generated.

The first characteristic is called cut-in speed, the minimum wind speed at which the wind turbine starts to generate usable power. The typical cut-in speed of a wind turbine is between 2 to 4 meters per second. As the wind speed rises greater than the cut-in speed, the output power (the blue curve) increases rapidly, until it reaches the limit that the wind turbine is capable of, typically at some speed between 12 to 17 meters per second. The limit is called rated output power and the corresponding wind speed is called rated speed. The output power rises no further and keeps at the constant level when the wind speed is greater than the rated speed. This characteristic forces the planner to treat the different ranges of wind speed separately when estimating the output power. The last characteristic is cut-out speed, at which turbines have to cease power generation and shut down, in order to protect them from damage when the wind speed is too high. Figure 1 shows an example of ENERCON turbine type E48 [18] with 800kW rated output power. The cut-in, rated and cut-out speed are 3, 14 and 25 meters per second, respectively.

### C. Gaussian Regression Based Evaluation Method

Different types of wind turbines have different cut-in speeds and/or rated output power/speeds. The output power of a type can be expressed by a piecewise function:

\[
P(v) = \begin{cases} 
0, & v \leq \nu^{\text{min}} \\
G(v), & \nu^{\text{min}} < v \leq \nu^{\text{rate}} \\
\nu^{\text{rate}}, & \nu^{\text{rate}} < v \leq \nu^{\text{out}} \\
0, & v > \nu^{\text{out}},
\end{cases}
\]  

where \(\nu^{\text{in}}\) (\(\nu^{\text{out}}\)) is the cut-in (cut-out) speed, \(\nu^{\text{rate}}\) (\(\nu^{\text{rate}}\)) is the rated speed (the rated output power), and \(G(v)\) is a fitted curve for the ramp-up. For example, the output power curve in Figure 1 consists of three pieces: the output power is 0 when the wind speed is lower than 3 m/s, 800 kW when higher than 14 m/s, and in-between is a ramp-up. Moreover, different turbine types have different relations between power coefficient and wind speed. Hence, we have to do output power curve fitting for each turbine type.

We tried multiple curve fitting methods for \(G(v)\), including polynomial regression with different degrees up to 9, exponential fitting, Gaussian Regression with different number of items up to 5. We use the data in the product sheet in [18] for training. Gaussian Regression was found out to have the least training error, i.e., root-mean-square error (RMSE) less than several watts. RMSEs for other fitting methods were at least 75% worse. Note that different turbine types of may have different optimal numbers (n in Equation (4)) of Gaussian items and we use the optimal number for each turbine type. The ramp-up curve \((G(v))\) is as follows:

\[
G(v) = a_1 e^{-\frac{(v-h)^2}{2s^2}} + \ldots + a_n e^{-\frac{(v-h)^2}{2s_n^2}}, 1 \leq n \leq 5,
\]

where \(a_n, b_n, c_n\) are parameters to be fitted.

Now we can calculate the output power \(P(v_t)\) of a wind turbine by Equation (3) with the wind speed \(v_t\) at each time \(t\). One way to assess the quantity of the power is called capacity factor, which is defined as the ratio of the actual output power to the maximum possible output or rated output power. In general, higher capacity factor means larger output power. The average capacity factor (\(\mu\)) during a time period \([0, T]\), for a given site and a given type of wind turbine, can be calculated as follows:

\[
\mu = \frac{1}{T} \sum_{t=0}^{T} \frac{P(v_t)}{\nu^{\text{rate}}}.
\]

Furthermore, the quality of the wind power for the site and the turbine type, can be interpreted as the standard deviation (\(\sigma\)) of the capacity factor:

\[
\sigma = \sqrt{\frac{1}{T} \sum_{t=0}^{T} \left(\frac{P(v_t)}{\nu^{\text{rate}}} - \mu\right)^2}.
\]

For clarity, we summarize the Gaussian Regression based evaluation method as follows:

### Gaussian Regression Based Evaluation Method

1. **Step 1** Adjust the wind speed according the turbine height by Equation (1);
2. **Step 2** Derive \(P(v)\) in Equation (3) by Gaussian Regression;
3. **Step 3** Calculate the average capacity factor and standard deviation (variance) by Equation (5)(6).
III. Locally Optimal Wind Power Generation

As discussed above, different types of wind turbines possess different characteristics on power conversion. Hence, there is some optimal type of wind turbine perfectly suitable to the statistical property of wind resource for a site. Those quantity-oriented methods would select the turbine type that generates the maximum average capacity factor. However, this would bring considerable additional cost for they overlook the quality. This section first shows the trade-off between the quantity and quality of wind power, and then addresses how to balance the trade-off with the focus on the optimal turbine type combination for a specific site. The next section will discuss further about this issue through exploring geographical diversity among different sites.

A. Ranking Turbines for A Site

We employ the real-world historical wind speed data extracted from National Solar Radiation Database (NSRD) [22], which contains wind speed field, solar irradiation field and other meteorological fields (such as sky cover, temperature, humidity). This database consists of a serially complete collection of hourly meteorological data from Jan 1st, 1991 to Dec 31th, 2010. NSRD contains the data from more than one thousand observation sites which are coded with USAF number (site ID). As to the data for wind turbines, ENERCON is one of the few companies who have published some detail parameters of their wind turbines, especially on the relation between the output power and wind speed. The data of a series of ENERCON products given in [18] is adopted here. We choose the maximum hub height for each type of the wind turbine when calculating the output power. Note that each type of wind turbine can be equipped with hubs of different heights (from tens to more than one hundred meters) [18], and different watts of wind power would be generated for different hub heights according to Equation (1)(3). The proposed approaches in this paper could be easily adapted to select both wind turbine types and hub heights. In addition, we do not consider how to deploy wind turbines for a specific area, but the proposed approaches could be combined with some existing works such as [30].

Based on the Gaussian Regression based evaluation method proposed in Section II-C, we rank different types of wind turbines for each site, according to their average capacity factor and standard deviation (or variance) respectively over the twenty years. Let us take a site with ID 723830 (Sandberg, CA) as an example to illustrate this. Table I shows the ranking result for the ten turbine types from [18]. One important observation here is that the turbine type with the maximum average capacity factor is different from the type with the minimum variance. For example, type E101 (3000kW) has the best average capacity factor but rather poor variance; while type E44 (900kW) is with the lowest average capacity factor but the best variance.

In addition, there is significant difference between the best and worst type in the output power’s quantity and quality. Type E44 (900kW) is more than 32% worse than type E101 (3000kW) at average capacity factor but nearly 24% better than type E82 (2300kW) at variance. Therefore, installing only one single type of turbine in a site can not achieve the optimal quantity and the optimal quality for wind power at the same time. Moreover, the turbine type with the largest rated output power is always not optimal at either quantity or quality. For instance, type E126 (7500kW) has neither the best average capacity factor nor the smallest variance. We did the similar ranking for all other sites in NSRD and these observations above are always true.

B. Optimal Combination of Turbine Types

According to the analysis above, we can conclude that one should install a combination of different types of wind turbines, in order to meet the requirements of both high average capacity factor and low variance for a site. In addition, we assume that there is little influence from other turbines on calculating the capacity factor for a turbine. The assumption is reasonable since the influence would be lowered as much as possible when deploying wind turbines in a large area of land. This combination problem can be split into two subproblems. Subproblem P1 is that, for a given requirement on the average capacity factor (M), to minimize the variance; Subproblem P2 is that, for a given requirement on the variance (\(\Delta\)), to maximize the average capacity factor. We formulate the two subproblems into Convex Programming (CP) problems as follows:

\[
P1: \min \sum_i \sum_j w_i w_j \text{Cov}(\frac{P_{i,v}}{P_{max}}, \frac{P_{j,v}}{P_{max}}) \\
\text{sub. to} \sum_i w_i \mu_i \geq M, \\
\sum_i w_i = 1, \\
0 \leq w_i \leq 1, \\
\]

\[
P2: \max \sum_i w_i \mu_i, \\
\text{sub. to} \sum_i \sum_j w_i w_j \text{Cov}(\frac{P_{i,v}}{P_{max}}, \frac{P_{j,v}}{P_{max}}) \leq \Delta, \\
\sum_i w_i = 1, \\
0 \leq w_i \leq 1, \\
(7)
\]

where \(w_i\) is the weight that the total rated output power of each turbine type holds in a combination, \(\text{Cov}(\frac{P_{i,v}}{P_{max}}, \frac{P_{j,v}}{P_{max}})\) is the covariance of capacity factor between every two turbine
types, and $\mu_i$ is the average capacity factor of each type. There are many efficient algorithms for solving CP problems, such as Interior-point methods [31]. Larger weight for a turbine type means selecting more turbines from the type, while zero weight indicates eliminating the type from the combination. Moreover, using the resulted weight, one can easily calculate the number turbines from each type to be installed for a required wind power capacity.

The fundamental concept behind the two subproblems is that the weight in a combination should not be selected individually on their own merits of each turbine type. Rather, it is important to consider how each type changes in quality (quantity) relative to how every other type in the combination changes in quality (quantity). The resulted combination strikes a balance between wind power’s quantity and quality. In general, the turbine type with higher quantity or larger average capacity factor is of lower quality or of more variance (See the example in Table I). Therefore, the two subproblems can be seen as a form of diversification. Under certain assumptions and specific quantitative constraints of quality (quantity), the convex problems in Equation (7) would find the best possible diversification strategy for a site.

C. Results and Analysis

The wind speed data of site 723830 is also used in the experiment of this section. To illustrate the effectiveness of the proposed approach (CP problems in (7)) at balancing the wind power’s quantity and quality, we compare them with other two approaches. One approach is the Uni-Type, by which only a single type of turbine is installed for a site. For problem $P_1$, the Uni-Type approach would select the turbine type that satisfies the capacity factor requirement and with the smallest variance; while for problem $P_2$, this approach would choose the turbine type meeting the variance requirement and with the maximum capacity factor. Uni-Type can be seen as a superset of those quantity-oriented methods. Let us give an example of problem $P_1$ using the Uni-Type approach. For a required capacity factor of 0.4, the first four turbine types in column 2 of Table I can meet the requirement. Hence, the Uni-Type approach will choose type E53(800kW), since it has the smallest variance among the four. One could reach a similar example for problem $P_2$ using Table I. The other approach is called Max-Rate, by which the turbine type with the maximum rated output power is selected for a site, e.g., type E126 (7500kW) in Table I. To the best of our knowledge, there is no existing work that addressed the turbine type combination problem and thus the Uni-Type and Max-Rate approach are used for comparison here.

Figure 2 shows the comparison results between CP (in Equation (7)), Uni-Type and Max-Rate approach. For problem $P_1$, CP significantly outperforms the other two approaches. Max-Rate, the red straight line in the figure, always has the worst performance, because it simply selects the turbine with the largest rated output power which is neither quantity-optimal nor quality-optimal. The curve is shorter than the other two, since type E126 (7500kW) can not satisfy the required capacity factor any more after the value of 0.37. Uni-Type performs better than Max-Rate, as it chooses the turbine type with the least variance after meeting the capacity factor requirement. Since the capacity factor for all considered turbine types is greater than 0.3 for site 723830 (see Table I), the CP and Uni-Type curves are flat before this value point. In addition, Uni-Type is a staircase curve. Each step represents a change in the selection of turbine type for the site and there are four times of this kind of change in total. Due to the optimal combination of weighted turbine types, CP provides a more smooth curve and has up to 21.9% lower variance than Uni-Type. Moreover, the two curves have the similar trend about the variance changing with the required capacity factor. This is due to the nature of wind resource, that is, larger capacity factor of a site always comes at the expense of higher variance in wind power generation.

Similar result analysis can be made for problem $P_2$ according to Figure 2-(b), such as shorter curve for Max-Rate and staircase curve for Uni-Type. We omit it here to avoid repetition. It’s worth noting that CP outperforms Uni-Type across all valid required variance, and has up to 32.2% more capacity factor than Uni-Type.

Insights: Different types of wind turbines can significantly compensate each other by smoothing each other’s fluctuation of the output power.

IV. GLOBALLY OPTIMAL WIND POWER GENERATION

Last section exploits the difference of power conversion characteristics between different turbine types and deals with the problem of turbine type combination. This is an effective
approach from the point of view of the owner of a single wind farm. However, the electrical grid is going to integrate more wind power from geographically distributed wind farms. The locally optimized turbine combination would not achieve a desired global result from the angle of the electrical grid. This section also addresses the quantity-quality trade-off problem, but further explores the geographical diversity among different sites, i.e., jointly optimizes the combination of sites as well as turbine types.

A. Globally Ranking Wind Power Generation

With the turbine ranking for all sites in NSRD [22], we observe that these sites have different optimal turbine types (or different rankings on turbine type) on both quantity and quality of wind power. For example, Table II shows the turbine type ranking on capacity factor for three sites with ID of 722524, 722953 and 723830. The quantity-optimal type for each site is type E82 (2000kW), E82 (2300kW) and E101 (3000kW) respectively. The reason is that different sites possess different statistical properties of wind resource due to the geographical diversity among them.

Another observation is that the three evaluation methods discussed in Section II produce rather different results on assessing the wind resource. This can be observed from Table III, which exhibits the ranking results for 9 sites by the three methods. The 9 sites are with complete wind speed trace and have the largest capacity factor (based on Gaussian evaluation method) among all of the more than one thousand sites recorded in NSRD. The table also demonstrates the misleading results of the first two methods at evaluating the wind resource for different sites. Let us compare the best site by each method. For the method only using the adjusted wind speed by Equation (1), the best is site 724510; for the method using wind power density by Equation (2), it is site 725745; for our Gaussian Regression based method, it is site 723630. They derive different best sites. Furthermore, the difference of ranking results between the three methods will become even larger when more sites are considered.

One more observation is that none of the sites possess the best average capacity factor and the smallest variance at the same time. For instance, site 723630 and site 723525 (at the rightmost two columns of Table III) have the best quantity and quality respectively. Therefore, the quantity-quality trade-off also exists in site selection.

B. Optimal Combination of Sites and Turbine Types

This subsection solves the two problems (P1 and P2) through exploiting both the geographical diversity and the difference of power characteristics between turbine types. The two kinds of diversities bring more chance to strike a balance on the trade-off and to achieve more cost effective solutions. Meanwhile, this also makes the problem become even harder. We have to deal with whether to choose a site in the solution or the weight for each site, in addition to the weight for each turbine type in the site. To avoid boolean variables in the program that represent whether a site is chosen or not, we set a weight variable ($w_{ij}$) for each turbine type $j$ in each site $i$. When to extract the sites’ combination in the solution, we only need to check the sum of the weight for the turbine types in each site. That is, for a site $i$, if $\sum_j w_{ij} \neq 0$, then the site is chosen; otherwise, the site is eliminated from the solution. The Convex Programming formulations for problem P1 and P2 are as follows:

\[
\text{P1 : } \min \sum_i \sum_j \sum_k \sum_l w_{ij} w_{kl} Cov(P_{ij}(w), P_{kl}(w)) \\text{sub. to } \sum_i \sum_j w_{ij} \mu_{ij} \geq M, \\
\sum_i \sum_j w_{ij} = 1, \\
0 \leq w_{ij} \leq 1,
\]

\[
\text{P2 : } \max \sum_i \sum_j w_{ij} \mu_{ij} \\text{sub. to } \sum_i \sum_j \sum_k \sum_l w_{ij} w_{kl} Cov(P_{ij}(w), P_{kl}(w)) \leq \Delta, \\
\sum_i \sum_j w_{ij} = 1, \\
0 \leq w_{ij} \leq 1,
\]

where $Cov(P_{ij}(w), P_{kl}(w))$ is the covariance of capacity factor between turbine type $j$ in site $i$ and turbine type $l$ in site $k$, and $\mu_{ij}$ is the average capacity factor of each turbine type $j$ in each site $i$. Using the resulted weight, one can easily check whether a site is selected and calculate the number of each type of turbine to be installed for an expected power capacity in the selected sites.

---

### Table II

<table>
<thead>
<tr>
<th>Rank</th>
<th>Sites</th>
<th>Capacity Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E82 (2000kW)</td>
<td>E82 (2000kW)</td>
</tr>
<tr>
<td>2</td>
<td>E101 (3000kW)</td>
<td>E101 (3000kW)</td>
</tr>
<tr>
<td>3</td>
<td>E53 (800kW)</td>
<td>E53 (800kW)</td>
</tr>
<tr>
<td>4</td>
<td>E82 (2300kW)</td>
<td>E82 (2300kW)</td>
</tr>
<tr>
<td>5</td>
<td>E53 (330kW)</td>
<td>E53 (330kW)</td>
</tr>
<tr>
<td>6</td>
<td>E82 (3000kW)</td>
<td>E82 (3000kW)</td>
</tr>
<tr>
<td>7</td>
<td>E48 (800kW)</td>
<td>E48 (800kW)</td>
</tr>
<tr>
<td>8</td>
<td>E126 (7500kW)</td>
<td>E126 (7500kW)</td>
</tr>
<tr>
<td>9</td>
<td>E70 (2300kW)</td>
<td>E70 (2300kW)</td>
</tr>
<tr>
<td>10</td>
<td>E44 (900kW)</td>
<td>E44 (900kW)</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Rank</th>
<th>Wind Speed</th>
<th>Cubic Power</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>724510</td>
<td>723525</td>
<td>725745</td>
</tr>
<tr>
<td>2</td>
<td>723630</td>
<td>723630</td>
<td>724510</td>
</tr>
<tr>
<td>3</td>
<td>723526</td>
<td>723526</td>
<td>725745</td>
</tr>
<tr>
<td>4</td>
<td>724515</td>
<td>727645</td>
<td>723525</td>
</tr>
<tr>
<td>5</td>
<td>725745</td>
<td>724515</td>
<td>723525</td>
</tr>
<tr>
<td>6</td>
<td>724585</td>
<td>724585</td>
<td>727645</td>
</tr>
<tr>
<td>7</td>
<td>727645</td>
<td>724515</td>
<td>723525</td>
</tr>
<tr>
<td>8</td>
<td>723525</td>
<td>725063</td>
<td>724515</td>
</tr>
<tr>
<td>9</td>
<td>725063</td>
<td>725745</td>
<td>725745</td>
</tr>
</tbody>
</table>
C. Results and Analysis

1) Statistical Results: The wind speed data of the 9 sites in Table III and Gaussian Regression based evaluation method in Section II-C are used in the experiment of this section. For comparison, we construct other three two-step methods: Uni-Type + Opt-Wgt, Max-Rate + Opt-Wgt and Opt-Type + Eql-Wgt. The sub-methods at the left side of the plus sign are first used to choose the turbine type, and then those at the right side are used to determine the weight for each site. The detail description for each sub-method is as follows:

- Uni-Type and Max-Rate: the same approaches as presented in Section III-C;
- Opt-Type: the optimized combination of turbine type by using the approach in Equation (7);
- Eql-Wgt: the approach that assigns each site with the equal weight in the total rated output power;
- Opt-Wgt: a similar approach to Equation (8), but only considering the covariance of capacity factor between different sites and not considering that between different turbine types.

Through comparing with these three approaches, we can clearly show the performance gain of the proposed method in Equation (8) at either of the two aspects: turbine combination and site combination.

Figure 3 shows the comparison results between CP, Max-Rate + Opt-Wgt, Uni-Type + Opt-Wgt and Opt-Type + Eql-Wgt. For problem P1, the performance of Max-Rate + Opt-Wgt (the red curve in the figure), degrades quickly after the required capacity factor of 0.33 and has the worst performance among the four methods. In addition, the curve is much shorter than others and stops before the required capacity factor reaches the value of 0.37. One part of the reason here is quite similar to those for Max-Rate in Section III-C. The other part is that the bad performance of Max-Rate dominates that of Opt-Wgt. Uni-Type + Opt-Wgt performs better than Max-Rate + Opt-Wgt, because the former approach, to some extent, considers to choose a better type of wind turbine at reducing the variance of the output power. Even involving the optimal site combination Opt-Wgt in the second step, the two approaches still come with very poor performance. This clearly indicates the crucial effect of turbine type combination at reducing the variance.

Another observation showing the above effect is that, compared with other three approaches whose curves are all monotonously increasing, Uni-Type + Opt-Wgt has a different trend about the variance changing with the required capacity factor. The Uni-Type + Opt-Wgt curve starts to decrease after the capacity factor becomes larger than 0.38, and then increases again after the capacity factor of 0.41. This is because of the fact that the best turbine types for each site usually would not coincide with the turbine types with the lowest variance for site combination.

We thus conducted the experiment for Opt-Type + Eql-Wgt. Although the performance of this method becomes slightly better than Uni-Type + Opt-Wgt after the capacity factor of 0.3, it still has rather larger variance of output power even with the optimized turbine type combination in the first step. This is because Eql-Wgt totally ignores the optimization on site combination. Therefore, in order to achieve the optimal variance, we should jointly consider the turbine type combination and site combination. This is what the proposed method CP does. From Figure 3-(a), we can see that CP performs best, and yields up to 34.5% lower variance than Uni-Type + Opt-Wgt and up to 18.4% than Opt-Type + Eql-Wgt (whose curve is also shorter). In addition, before the required capacity factor of 0.3, all curves are flat. There are two reasons for this. One is that the capacity factor for each site and each turbine type is larger than 0.3; the other one is that the turbine type chosen for each of the four approaches stay unchanged and the weight of the types for each site also keeps the same when the required capacity factor is less than 0.3.

Similar result analysis can be made for problem P2 according to Figure 8-(b), such as CP outperforms all other approaches and Max-Rate + Opt-Wgt performs worst. There are two additional points that need to be noted here. The first one is that Uni-Type + Opt-Wgt becomes the shortest curve among all the four in this case. This approach for problem P2 first selects the turbine type that satisfies the required variance by Uni-Type. However, there is no single type of turbine that can satisfy the required variance lower than 0.09. So the curve stops at this variance value. The second point is that CP has the longest curve. The approach can deal with very tough requirements on the variance of output power and can find
we mainly focus on the analysis for the trend about capacity factor/variance varying with the number of candidate sites in this subsection. For problem $P_1$, shown in Figure 4-(a), as the number of candidate sites increases, the resulted variance for a specific requirement on capacity factor decreases. The reason is that it is scarcely possible that all candidate sites have no wind or zero wind speed at the same time. As more sites are combined, there is more possibility for these sites to compensate each other in power generation. To the extreme, we may expect the case that the variance of output power is equal to zero if perfectly combining a large amount of sites. However, from the figure, we can see that the decreasing rate of the variance becomes smaller and smaller as the number of candidate sites grows. To realize the perfect case with zero variance, a large amount of very small-scale wind farms in different sites need to be constructed for a specific required capacity of output power. This is neither realistic nor cost effective. Hence, how to choose an optimal number of wind farms to achieve both balanced quantity-quality and cost effectiveness is not a trivial problem, and we leave it as our future work.

Similarly, for problem $P_2$, the resulted capacity factor for a specific requirement on variance increases as the number of candidate sites grows. One observation that needs to be noted here is that selecting just a few number of sites could meet a lower requirement on the variance of the output power. For example, as shown by the dark red part of the mesh in Figure 4-(b), for required variances greater than 0.06, the resulted capacity factors are nearly the same when combining more than three sites.

**Insights:** Though combining more sites comes with less variance, it needs to combine only a few number of sites to meet not too stringent requirements on quality.

### D. Discussion

More consideration should be taken for site selection when to construct wind farms in the smart grid. Besides the cost for wind turbines, there are several other costs, such as electricity grid connection, land purchase, infrastructure, and so on. Therefore, it is not cost effective to construct many wind farms in very small scale just to meet the requirement on the capacity factor or variance of output power as mentioned above. That is, the resulted weight from the proposed CP in Equation (8) should be not too small. To guarantee a proper scale for each wind farm to be constructed, we can first add some constraints to the problem, such as the weight for each site must be greater than a threshold value $w_{\text{min}}$. However, if there are a large amount candidate sites, still $1/w_{\text{min}}$ sites may be picked out for a resulted site combination. For example, 100 sites may be chosen when $w_{\text{min}} = 0.01$. Then, a constraint to limit the number of selected sites should be adopted then. This point is quite different with that for turbine type selection, where the weight for each turbine type to be installed could be rather small and there is no need to limit the number of the types. This constraint could also guide us to figure out the cost-effective number of wind farms that need to be constructed.
We will deal with the P1 and P2 problem with the above two additional constraints in more detail in our future work, where an detail analysis about the relation of the threshold weight and the number of selected sites with the expected capacity factor and variance, will be provided.

V. CONCLUSION

The cost of electricity from renewable energy sources would rise, since deep integration of them needs many additional operations for the electrical grid. Numerous advancements in technologies and methods are required for the smart grid. This paper focuses one of them: how to construct wind farms of large quantity and high quality in the smart grid. Using the proposed Gaussian Regression based evaluation method, we analyzed the trade-off between wind power’s quantity and quality. A novel approach was then proposed for optimally combining different types of wind turbines under the requirement of either the capacity or variance. Finally, the approach was extended to jointly optimize the combination of sites and turbine types by exploring the geographical diversity among different sites. Encouraging experimental results were shown for both of the two approaches and several interesting findings were also discussed. For the future work, to make a complete framework, we will first take account of more real-world factors into this wind farms’ construction planning problem, including decision of optimal number of sites, optimization of the overall cost, power loss on transmission and energy storage systems. Then, the proposed approaches in this paper will be adapted to combine other kinds of renewable energy resources such as solar power [32], [33] and hydrogen power [34].

Acknowledgement: We would like to thank the anonymous reviewers for their constructive comments. This work was supported in part by the NSERC Discovery Grant 341823 and McGill Tomlinson Scientist Award.

REFERENCES