Assessment of Multi-Hop Interpersonal Trust in Social Networks by Three-Valued Subjective Logic

Guangchi Liu*, Qing Yang*, Honggang Wang†, Xiaodong Lin‡ and Mike P. Wittie∗

*Department of Computer Science, Montana State University, Bozeman, MT, USA
†Department of Electrical and Computer Engineering, University of Massachusetts Dartmouth, North Dartmouth, MA, USA
‡Faculty of Business and Information Technology, University of Ontario Institute of Technology, Oshawa, Ontario, CA

Abstract—Assessing multi-hop interpersonal trust in online social networks (OSNs) is critical for many social network applications such as online marketing but challenging due to the difficulties of handling complex OSN topology, in existing models such as subjective logic, and the lack of effective validation methods. To address these challenges, we for the first time properly define trust propagation and combination in arbitrary OSN topologies by proposing 3VSL (Three-Valued Subjective Logic). The 3VSL distinguishes the posteriori and priori uncertainties existing in trust, and the difference between distorting and original opinions, thus be able to compute multi-hop trust in arbitrary graphs. We theoretically proved the capability based on the Dirichlet distribution. Furthermore, an online survey system is implemented to collect interpersonal trust data and validate the correctness and accuracy of 3VSL in real world. Both experimental and numerical results show that 3VSL is accurate in computing interpersonal trust in OSNs.

Index Terms—Interpersonal trust, trust establishment, online social networks, three-valued subjective logic, Dirichlet distribution

I. INTRODUCTION

With the emergence of Online Social Networks (OSNs), user-generated content and interactions have dominated web activity and the questions of whom and what to trust have become increasingly important. Trust assessment in OSNs can be applied in various domains such as online marketing, proactive friendships construction [1], and networking security [2]–[8]. Various online trust models have been developed but one thing missing from them is that they do not have an interpersonal trust component, which widely exists in real world. Therefore, this paper tries to answer a fundamental question: how to accurately model and compute the multi-hop interpersonal trust between two users connected within OSNs?

Trust can be defined in different ways such as rating-based reputation in online marketing and local search applications, preferences in recommendation systems, and probability of being a normal node in network security analysis. We define interpersonal trust as the probability that a trustee will behave as expected by a trustor. This general definition of interpersonal trust makes it applicable for a wide range of applications. We also assume trustworthiness is determined by objective evidences, i.e. the cognitive features of trust are not our concern [9]. Subjective logic models trust as a tuple in the form of (belief, distrust, uncertainty) and is widely used in assessing multi-hop interpersonal trust [10]. However, it can only handle series-parallel social network topology, i.e. complex topology needs to be simplified by removing or selecting edges. Such simplification will result in information loss and inaccurate trust assessment, which is confirmed in our numerical analysis.

To cope with arbitrary topology in OSNs, we propose the 3VSL (Three-Valued Subjective Logic) as a solution for assessing multi-hop interpersonal trust in OSNs. Unlike subjective logic, we define interpersonal trust as a trinary event (belief, distrust, neutral) instead of a binary event (belief, distrust), hence extend Beta distribution to Dirichlet distribution. Neutral state expresses the posteriori uncertainty in trust generated by trust propagation, which is ignored in subjective logic. The introduction of neutral state makes the operations in 3VSL different from subjective logic. Leveraging on this new definition, operations (discounting and combining) on trust are redesigned in 3VSL. Furthermore, a algorithm of applying 3VSL to compute trust in arbitrary social network topology is given and proved to be correct. To the best of our knowledge, 3VSL is the first model of computing multi-hop interpersonal trust in arbitrary network topology.

Also, we implement an online survey system to collect interpersonal trust data. 100 participants are invited to evaluate the trustworthiness of their 1st and 2nd hop friends by filling out a questionnaire designed for measuring interpersonal trust [11]. In addition, we conduct a numerical analysis to show the features of 3VSL. Experimental and numerical results indicate that 3VSL is accurate in assessing interpersonal trust.

Major contributions of this paper are as follows:
1. The shortcomings of subjective logic-based approaches are identified and addressed in the 3VSL model. 3VSL distinguishes the priori and posteriori uncertainties, the distorting and original opinions, and redefines the discounting and combining operations on opinions. 3VSL is proved to be able to assess multi-hop interpersonal trust in arbitrary network topology.
2. The 3VSL model is validated by the interpersonal trust data collected from 100 participants. To the best of our knowledge, this is the first established dataset for assessing interpersonal trust in real-life multi-hop social networks.

II. RELATED WORK

Many previous efforts have been devoted to the study of using probability distribution to express and model trust in a computational way. In probability distribution-based methods,
trust is defined as a probability distribution of a certain event. For example, Beta distribution [10] is used to model binary discrete random variables, i.e. whether a person is trustworthy or not. In [12], [13], Gaussian distribution is proposed to handle the situation where possible outcomes are continuous random variables. Beta distribution is then extended to the Dirichlet distribution to handle multiple discrete random variables [14]. Probability distributions are often associated with Bayesian analysis [12], [13] so that they can model trust by integrating information from various sources such as reputations, preference and group behaviors.

Based on Beta distribution, Josang, etc. proposed subjective logic [10] to assess interpersonal trust through trust propagation and fusion, among people who do not have direct social connections between each others. In [15], [16], how to apply subjective logic in trust graph is studied. Subsequently, it is redefined in [17]–[20] and the accuracy of trust estimation is improved. In [21], the authors introduce the selection operation which selects the strongest path to compute multi-hop trust when several trust paths exist in social networks. In [22], trust information computed by subjective logic is used in the routing domain.

Nevertheless, all previous work are not able to correctly define how interpersonal trust changes during trust propagation. Due to this issue, subjective logic cannot handle complex topology in social networks. Unlike subjective logic, 3VSL considers certain evidences are distorted and transferred into the neural state when trust propagates from one person to another. As a complementary of one-hop trust estimation, we believe 3VSL is an improvement to the framework of social trust computation.

III. SYSTEM MODEL AND PROBLEM STATEMENT

A trust social network is modeled as a directed graph $G(V, E)$ where a vertex $u \in V$ represents a person, and an edge $e(u, v) \in E$ denotes how much $u$ trusts $v$ (from his/her direct interaction experience). Each edge is expressed as an opinion, which means $v$’s trustworthiness to $u$.

In a trust social network, two edges are in series if they are incident to a vertex of degree 2 and are parallel if they join the same pair of distinct vertices. As shown in Fig. 1(a) and Fig. 1(b), two users can be simply connected in a serial topology or parallel topology. They can also be connected in a bridge topology (in Fig. 1(c)), where the connection from $A$ to $C$ cannot be decomposed into series and parallel topologies.

To apply subjective logic, the graph between two users must be a directed series-parallel graph, i.e. subjective logic only works on directed two-terminal series-parallel graphs (DTTSPG) [23]. Since the bridge topology is not a DTTSPG, it is unsolvable in subjective logic. Although approximation algorithms were proposed [16], [21], information loss was found in the process of selecting and removing edges.

In fact, the bridge (or even arbitrary) topology is solvable in 3VSL by distinguishing the distorting opinion ($AB$) and original opinion ($DC$), which will be discussed in the following sections. Since bridge topology is common in social networks, it is important to design a model to compute the trustworthiness between users connected by a bridge or arbitrary topology without losing information. Formally, we define the problem of computing interpersonal trust in social networks as follows:

**Problem Statement**

Given an arbitrary trust social network $G(V, E)$, $\forall u$ and $v$ s.t. $e(u, v) \notin E$ and $\exists$ at least one path from $u$ to $v$, how to compute the trustworthiness of $v$ to $u$, i.e. how should $u$ trust a stranger $v$ based on her existing social connections.

IV. THREE-VALUED SUBJECTIVE LOGIC

To compute interpersonal trust in any arbitrary topology, we propose the Three-Valued Subjective Logic (3VSL) by distinguishing 1) the posteriori and priori uncertainties, and 2) distorting and original opinions. In 3VSL, the trustworthiness/opinion of a person is considered as a trinary event (true, false, neutral). Neutral state, also called posteriori uncertainty, keeps the the evidences distorted from certain spaces when trust propagates from one person to another. Leveraging on this new definition of trust, we redesign the discounting and combining operations on opinions, and theoretically prove that 3VSL is capable to handle arbitrary network topology.

A. Subjective Logic

To better understand 3VSL, we first briefly introduce the subjective logic [10]. Considering two users $A$ and $X$, the trustworthiness of $X$ to $A$ can be described by an opinion vector $\omega_X^A$:

\[
\omega_X^A = (b_X^A, d_X^A, u_X^A, a_X^A)
\]

\[
b_X^A + d_X^A + u_X^A = 1
\]

where $b_X^A$, $d_X^A$, $u_X^A$ and $a_X^A$ refer to the belief, distrust, uncertainty, and base rate. $a_X^A$ is a constant formed from an existing impression without solid evidences, e.g. prejudice, preference and general opinion obtained from hearsay. For example, if $A$ always distrusts/trusts the persons from a certain group where $X$ belongs to, then $a_X^A$ will be smaller/greater than 0.5. Based
on the Beta distribution, two opinions $\omega_1 = (b_1, d_1, u_1, a_1)$ and $\omega_2 = (b_2, d_2, u_2, a_2)$ can be combined as follows:

$$
\begin{align*}
    b_{12} &= \frac{b_1 u_2 + b_2 u_1}{u_1 + u_2 - u_1 u_2} \\
    d_{12} &= \frac{d_1 u_2 + d_2 u_1}{u_1 + u_2 - u_1 u_2} \\
    u_{12} &= \frac{u_1 + u_2 - u_1 u_2}{u_1 + u_2 - u_1 u_2} \\
    a_{12} &= a_2 
\end{align*}
$$

Let $A$ and $B$ be two persons where $\omega^A_B = (b_1, d_1, u_1, a_1)$ is $A$’s opinion about $B$’s trustworthiness, and let $C$ be another person where $\omega^C_B = (b_2, d_2, u_2, a_2)$ is $B$’s opinion about $C$. Then, subjective logic applies the discounting operation to compute $\omega^C_B$ as follows.

$$
\begin{align*}
    b_{12} &= b_1 b_2 \\
    d_{12} &= b_1 d_2 \\
    u_{12} &= 1 - b_{12} - d_{12} - u_2 \\
    a_{12} &= a_2 
\end{align*}
$$

Finally, the expected belief of an opinion $\omega_X^A$ is computed by $E(\omega_X^A) = b_X^A + u_X^A a_X^A$.

B. Opinion Vector in 3VSL

In 3VSL, an opinion vector $\omega_X^A$ is defined as:

$$
\omega_X^A = (b_X^A, d_X^A, n_X^A, e_X^A) | a_X^A = 1
$$

where $b_X^A$, $d_X^A$, $n_X^A$, $e_X^A$ and $a_X^A$ refer to belief, distrust, posteriori uncertainty, and priori uncertainty, and base rate. $b_X^A$, $d_X^A$, $n_X^A$ represent the probabilities that $X$ is trustworthy, not trustworthy, and neutral, respectively. $e_X^A$ represents priori uncertainty (without evidences) of whether $X$ is trustworthy, not trustworthy, or neutral. Priori uncertainty exists due to the lack of evidences, while posteriori uncertainty exists because of evidence distortions. The definition of $a_X^A$ is the same as that in subjective logic and it keeps unchanged in 3VSL, for simplicity we will not mention it unless necessary.

As shown in Fig 2, the certainty of an opinion comes from $b_X^A$ and $d_X^A$, while uncertainty from $n_X^A$ and $e_X^A$. For example, if $A$ has no interaction with $X$, then her opinion on the trustworthiness of $X$ is $(0, 0, 0, 1)$. Later on, if $A$ receives 1, 2 and 4 evidences to support $X$ is trustworthy, not trustworthy and neutral, then $\omega_X^A$ becomes $(0.1, 0.2, 0.4, 0.3)$. The details of how to compute $e_X^A$ will be shown in the following sections. Compared to subjective logic, 3VSL further distinguishes the uncertainty as priori and posteriori.

C. Dirichlet Distribution in 3VSL

To support mathematical operations on opinion vectors in 3VSL, we first define a mapping between the probability density function (pdf) representation and the opinion representation by introducing a evidence space. As shown in Fig. 3, the operations on opinions can be considered as the operations on the Dirichlet distributions.

The Dirichlet distribution is a family of continuous multivariate probability distributions parametrized by a vector $\alpha = < \alpha_1, \ldots, \alpha_K >$. Its pdf returns the belief that the probabilities of $K$ rival events are $x_i$ given that each event has been observed $\alpha_i - 1$ times. Since evidences in an opinion vector are in three possible states (belief, distrust, neutral), we use a trinomial Dirichlet distribution:

$$
f(P_b, P_d | \alpha, \beta, \gamma) = \frac{\Gamma(\alpha + \beta + \gamma)}{\Gamma(\alpha) \cdot \Gamma(\beta) \cdot \Gamma(\gamma)} \cdot P_b^{\alpha-1} \cdot P_d^{\beta-1} \cdot P_n^{\gamma-1}
$$

where $< \alpha, \beta, \gamma >$ is the controlling vector, $P_b$, $P_d$ and $P_n$ represents the probability of belief, distrust, and neutral events.

Let $r$, $s$ and $o$ be the number of evidences observed to support a person is trustworthy, not trustworthy, and neutral, respectively. According to the definition of the Dirichlet distribution, we have $\alpha = r + 1$, $\beta = s + 1$, and $\gamma = o + 1$.

Here we assume that a person already had one evidence of each event (belief, distrust and neutral). The assumption is reasonable because the Dirichlet distribution still works even when no event is observed, i.e. $(\alpha = 1, \beta = 1, \gamma = 1)$ and the probability of each event will be $1/3$. These three priori evidences are considered as priori uncertainty.

According to the definition of $\omega_X^A$, its four components can be expressed as:

$$
\begin{align*}
    b_X^A &= \frac{r}{r + s + o + 3} \\
    d_X^A &= \frac{s}{r + s + o + 3} \\
    n_X^A &= \frac{o}{r + s + o + 3} \\
    e_X^A &= \frac{3}{r + s + o + 3}
\end{align*}
$$

where the amount of priori evidences is set as 3, and its ratio to the number of observed evidences is $e_X^A$. Note that the uncertainty $u$ defined in subjective logic is actually the priori uncertainty $e$ in 3VSL.

Since the expected probability of each event in Eq. 1 is,

$$
\begin{align*}
    E(P_b) &= \frac{\alpha}{\alpha + \beta + \gamma} = \frac{r + 1}{r + s + o + 3} \\
    E(P_d) &= \frac{\beta}{\alpha + \beta + \gamma} = \frac{s + 1}{r + s + o + 3} \\
    E(1 - P_b - P_d) &= \frac{\gamma}{\alpha + \beta + \gamma} = \frac{o + 1}{r + s + o + 3}
\end{align*}
$$

we have the following equations:

$$
\begin{align*}
    E(P_b) &= \frac{r + 1}{r + s + o + 3} = b_X^A + \frac{1}{3} e_X^A \\
    E(P_d) &= \frac{s + 1}{r + s + o + 3} = d_X^A + \frac{1}{3} e_X^A \\
    E(1 - P_b - P_d) &= \frac{o + 1}{r + s + o + 3} = n_X^A + \frac{1}{3} e_X^A
\end{align*}
$$

where the probability of each event is determined by certain posterior evidences and priori evidences. So far we have built the mapping between opinion and the Dirichlet distribution by introducing an evidence space $(r, s, o)$.

Although priori uncertainty in 3VSL is similar to $u$ in subjective logic, posteriori uncertainty is considered a parallel state to distrust and belief. Posteriori uncertainty, belief and distrust form trinary events, so Beta distribution in subjective logic is extended to Dirichlet distribution in 3VSL.
in a parallel topology as shown in Fig. 1(b), opinions from parallel paths should be combined or fused in a fair and equal way so that the resulting opinion reflects all opinions. We first introduce the theorem of combining two opinions and then generalize it to support multiple opinions.

**Theorem 1** Let \( \omega_1 = (b_1, d_1, n_1, e_1) \) and \( \omega_2 = (b_2, d_2, n_2, e_2) \) be the opinions on two parallel paths between two users, then the combining operation \( \Theta(\omega_1, \omega_2) \) is carried out as follows:

\[
\Theta(\omega_1, \omega_2) = \begin{cases} 
  b_{12} = \frac{e_2 b_1 + e_1 b_2}{e_1 + e_2 - e_1 e_2}, \\
  d_{12} = \frac{e_2 d_1 + e_1 d_2}{e_1 + e_2 - e_1 e_2}, \\
  n_{12} = \frac{e_2 n_1 + e_1 n_2}{e_1 + e_2 - e_1 e_2}, \\
  e_{12} = \frac{e_1 + e_2 - e_1 e_2}{e_1 + e_2 - e_1 e_2}.
\end{cases}
\]

**Proof** According to the mapping relationship between opinion model and Dirichlet distribution, \( \omega_1 \) and \( \omega_2 \) can be represented as \( f(P_b, P_d | \alpha_1, \beta_1, \gamma_1) \) and \( f(P_b, P_d | \alpha_2, \beta_2, \gamma_2) \), respectively. If posterior evidences are independent, these two Dirichlet distributions can be aggregated into one:

\[
f(P_b, P_d | \alpha_1 + \alpha_2 - 1, \beta_1 + \beta_2 - 1, \gamma_1 + \gamma_2 - 1)
\]

It was shown that posterior evidences from the same state are accumulated except for the prior evidences. 3VSL considers the priori uncertainty from different individuals as identical and dependent. Then, the expected probability of belief in the aggregated distribution is:

\[
E(P_b) = \frac{\alpha_1 + \alpha_2 - 1}{\alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \gamma_1 + \gamma_2 - 3} = \frac{r_1 + r_2 + 1}{r_1 + r_2 + s_1 + s_2 + o_1 + o_2 + 3}
\]

Similarly, the expected probabilities of other events can be computed. We can see the number of posterior evidences of each events are increased to \( r_1 + r_2, s_1 + s_2, \) and \( o_1 + o_2. \) However, the amount of priori evidences is still 3 and does not change. 3VSL considers posteriori evidences of different single hop opinions are independent while priori evidences are dependent.

According to Eq. 1 and the definition of opinion vector, we have:

\[
b_1 = \frac{r_1}{r_1 + s_1 + o_1 + 3} = \frac{e_2 b_1}{(1 - e_1) + e_1 b_1}
\]

\[
b_2 = \frac{r_2}{r_2 + s_2 + o_2 + 3} = \frac{e_2 b_2}{(1 - e_1) + e_1 b_2}
\]

\[
e_1 \text{ and } e_2 \text{ are different if } r_1 + s_1 + o_1 \neq r_2 + s_2 + o_2, \text{ so we normalize them in the above equations. Combining these two opinions yields:}
\]

\[
b_{12} = \frac{e_2 b_1 + e_1 b_2}{(1 - e_1) + e_1 b_1 + (1 - e_2) + e_2 b_2} = \frac{e_2 b_1 + e_1 b_2}{e_1 + e_2 - e_1 e_2}
\]

Similarly, other components of the resulting opinion can be calculated as follows:

\[
d_{12} = \frac{e_2 d_1 + e_1 d_2}{e_1 + e_2 - e_1 e_2}, \quad n_{12} = \frac{e_2 n_1 + e_1 n_2}{e_1 + e_2 - e_1 e_2}, \quad e_{12} = \frac{e_1 + e_2 - e_1 e_2}{e_1 + e_2 - e_1 e_2}
\]

According to Theorem 1, we could expect the combining operation is both commutative and associative, i.e. \( \Theta(\omega_1, \omega_2) = \Theta(\omega_2, \omega_1) \) and \( \Theta(\omega_1, \Theta(\omega_2, \omega_3)) = \Theta(\Theta(\omega_1, \omega_2), \omega_3) \).

If there exist multiple parallel opinions \( \omega_1, \omega_2, \cdots, \omega_n \) between two users, the overall opinion can be calculated as \( \Theta(\Theta(\omega_1, \omega_2), \cdots, \omega_n) \). Since combining operation is commutative and associative, it can be simplified as \( \Theta(\omega_1, \omega_2, \cdots, \omega_n) \). Compared to subjective logic, new operation rules on priori and posterior uncertainties are introduced in 3VSL.

**E. Discounting Opinion in 3VSL**

The purpose of discounting opinion is to model trust propagation along trust connections within a social network. We first introduce how to discount one opinion from another, and then generalize it to support multiple opinions. Considering a simple case where \( A \) trusts \( B \) who trusts \( C \), the discounting operation allows \( A \) discounts \( B \)'s opinion over \( C \) to obtain her own opinion on \( C \).

**Definition 1 (Discounting Operation)** Let \( A \) and \( B \) be two persons where \( \omega^A_B = (b_1, d_1, n_1, e_1) \) is \( A \)'s opinion about \( B \)'s trustworthiness, and let \( C \) be another person where \( \omega^B_C = (b_2, d_2, n_2, e_2) \) is \( B \)'s opinion about \( C \). Then, the discounting operation \( \Delta(\omega^A_B, \omega^B_C) \) is carried out as follows:

\[
\Delta(\omega^A_B, \omega^B_C) = \begin{cases} 
  b_{12} = b_1 b_2, \\
  d_{12} = b_1 d_2, \\
  n_{12} = 1 - b_{12} - d_{12} - e_2, \\
  e_{12} = e_2
\end{cases}
\]

where \( \Delta(\omega^A_B, \omega^B_C) \) is \( A \)'s opinion about \( C \) as a result of \( B \)'s advice to \( A \).
Since $A$ discounts $B$’s opinion of $C$ to obtain her own opinion upon $C$, some certain evidences from $\omega_B^1$ will be distorted since $A$ does not absolutely trust $B$ and his opinion. The distorted evidences from $r_B^1$ and $s_B^1$ will be saved into the posteriori uncertainty space of the resulting opinion $\omega_A^1$. However, the priori evidences will be kept unchanged. As shown in Fig. 4, distorted evidences from belief and distrust spaces of $\omega_B^1$ are saved into the posteriori uncertainty space, resulting the same amount of evidences.

Discounting operation is analogical to electromagnetic wave propagation where original signal is distorted into a weak one at the receiving side. Trust propagating from $A$ to $C$ is considered as the opinion $\omega_C^B$ being distorted by $\omega_A^B$ (in the inverse direction). Since certain evidences from $\omega_B^1$ are distorted and saved into the posteriori uncertainty space of $\omega_A^1$, $\omega_B^1$ and $\omega_B^1$ have the same amount of evidences.

**Definition 2 (Distorting and Original Opinions)** Given a discounting operation $\Delta(\omega_1, \omega_2)$ on two opinions $\omega_1$ and $\omega_2$, we treat $\omega_1$ as the distorting opinion, and $\omega_2$ the original opinion. Since certain evidences from $\omega_2$ are distorted by $\omega_1$ and transferred into the posteriori uncertainty space of $\omega_2$, the evidence space of opinion $\omega_1, \omega_2$ is the same as $\omega_2$’. Therefore, we can conclude the resulting opinion of a discounting operation shares exactly the same evidence space with the original opinion. It is easy to prove that discounting operation is associative but not commutative, i.e. $\Delta(\omega_1, \omega_2) \neq \Delta(\omega_2, \omega_1)$ but $\Delta(\Delta(\omega_1, \omega_2), \omega_3) \equiv \Delta(\omega_1, \Delta(\omega_2, \omega_3))$. Given a serial topology where opinions are ordered as $\omega_1, \omega_2, \ldots, \omega_n$, the final opinion can be calculated as $\Delta(\Delta(\Delta(\Delta(\ldots, \omega_1, \omega_2), \ldots), \omega_n))$. Since discounting operation is associative, it is simplified as $\Delta(\omega_1, \omega_2, \ldots, \omega_n)$.

Compared to subjective logic, posteriori uncertainty is introduced in 3VSL to store neutral evidences eliminated from certainty spaces as trust propagates while priori uncertainty is kept unchanged.

**F. Difference between Distorting and Original Opinions in 3VSL**

To further understand the difference between distorting and original opinions, we investigate two special cases shown in Fig. 5. By analyzing these two topologies, we discover the shortcomings of subjective logic, and point out original opinions can be used only once in trust computation while distorting opinions are not.

**Lemma 1** Let $\omega_1, \omega_2, \omega_3$ be three opinions, then $\Delta(\omega_1, \Theta(\omega_2, \omega_3)) \equiv \Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$. However, $\Delta(\Theta(\omega_1, \omega_2), \omega_3) \neq \Theta(\Delta(\omega_1, \omega_3), \Delta(\omega_2, \omega_3))$.  

**Proof** We first prove $\Delta(\omega_1, \Theta(\omega_2, \omega_3)) \equiv \Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$. Let $\omega_i = (b_i, d_i, e_i, n_i)$ where $i = 1, 2, 3$, then both $\Delta(\omega_1, \Theta(\omega_2, \omega_3))$ and $\Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$ yield the same result: 

$$
\begin{align*}
  b &= b_1b_2e_3 + b_1b_3e_2 \\
  d &= b_1d_2e_3 + b_2d_3e_2 \\
  e &= e_2e_3 - e_3e_2
\end{align*}
$$

where

$$
\begin{align*}
  n_2 &= 1 - b_1b_2 - b_1d_2 - e_2 \\
  n_3 &= 1 - b_1b_3 - b_1d_3 - e_3
\end{align*}
$$

Now we prove $\Delta(\Theta(\omega_1, \omega_2), \omega_3) \neq \Theta(\Delta(\omega_1, \omega_3), \Delta(\omega_2, \omega_3))$. According to the aggregation rule defined in Dirichlet distribution, two Dirichlet distributions can be combined if and only if their evidence spaces are independent. Since opinions $\Delta(\omega_1, \omega_3)$ and $\Delta(\omega_2, \omega_3)$ share the same evidence space from the opinion $\omega_3$, they cannot be combined. Therefore, $\Theta(\Delta(\omega_1, \omega_2), \omega_3)$ is the only correct solution, and is not equal to $\Theta(\Delta(\omega_1, \omega_1), \Delta(\omega_2, \omega_3))$.

In subjective logic, $\Delta(\omega_1, \Theta(\omega_2, \omega_3))$ and $\Theta(\Delta(\omega_1, \omega_2), \Delta(\omega_1, \omega_3))$ will give different results, which is contradictory to the common sense. From Lemma 1, we also note that reusing $\omega_1$ is allowed, but not $\omega_3$. The difference between $\omega_1$ and $\omega_3$ is that $\omega_1$ is a distorting opinion while $\omega_3$ is an original opinion. Therefore, we conclude that in trust computation, original opinions can be combined only once, while distorting opinions can be combined any number of times because they do not change the amount of evidences in the final opinion.

**G. Expected Belief in 3VSL**

Given a opinion $\omega_A^X$, we need to calculate the expected probability that $A$ is affirmative that $X$ will perform the desired actions. Opinion $\omega_A^X$ contains four component $(b_A^X, d_A^X, e_A^X, n_A^X)$ which corresponds to vector in the evidence space $<r_A^X, s_A^X, o_A^X, 3 >$. In this vector, the expected number of positive evidences are $(r_A^X + a_X^A o_A^X + 1.5)$ where $a_X^A$ is the
By applying discounting and combining operations, the overall opinion $\omega^A_C$ of $A$ upon $C$ is solvable and unique.

**Proof** We prove the theorem in a recursive manner, i.e. reducing the original problem into sub-problem(s) and keep reducing the sub-problems until the base case is solvable and the solution is unique.

As shown in Fig. 7, we assume there are $m$ nodes $(c_1, c_2, \ldots, c_m)$ connecting to $C$, i.e. $e(c_i, C) \in E$ where $i \in [1, m]$. There are $n$ nodes $(a_1, a_2, \ldots, a_n)$ being connected from $A$, i.e. $e(A, a_j) \in E$ where $j \in [1, n]$.

**Reduction**

*Case 1:* If there is only one node connecting to $C$, i.e. $m = 1$, then $\omega^A_C = \Delta(\omega^A_{c_1}, \omega^A_C)$.

*Case 2:* If there are more than one node connecting to $C$, i.e. $m > 1$, based on Lemma 1, $\omega^A_C = \Theta(\Delta(\omega^A_1, \omega^A_C), \Delta(\omega^A_2, \omega^A_C), \ldots, \Delta(\omega^A_m, \omega^A_C))$. It is shown that $\omega^A_C$ is solvable and unique if and only if the $\omega^A_{c_i}$ is solvable and unique where $\omega^A_{c_i}$ is the result of the sub-problem with sub-graph $G' = G - \Sigma e(c_i, C) - C$.

For Case 2, the network topology from $\{a_j\}$ to $\{c_i\}$ is unknown and maybe complex, it is possible that all $c_i$ are connected at a certain node $b$. If $b = A$, then $\hat{A}bcC$ are parallel and can be combined. If $b \neq A$, $\omega^A_{c_1}$ can be computed in as 1) the combination of $\hat{A}bcC$, or 2) combining $bcC$ first and discounting it by $\omega^A_b$. According to Lemma 1, both methods yields the same result, i.e. although we use the first method in Algorithm 1, the final result is unique.

After each reduction, $G$ is reduced into $G'$ in the way that $|E| = |E| - m$ and $|V| = |V| - 1$. Applying reductions on sub-problems recursively, finally the base case will arise, i.e. $|E| = 1$ and $|V| = 2$.

**Base Case**

The sub-graph of base case contains only one edge from $A$ to $a_j$ where $j \in [1, n]$.

Since $\omega^A_{a_j}$ is known from the original graph $G$, the base case is solvable and the result is unique. Applying the equations in Case 1 and 2 repeatedly, we can obtain an unique $\omega^A_C$ of the original problem.

**Corollary** We prove the theorem in a recursive manner, i.e. reducing the original problem into sub-problem(s) and keep reducing the sub-problems until the base case is solvable and the solution is unique.

As shown in Fig. 7, we assume there are $m$ nodes $(c_1, c_2, \ldots, c_m)$ connecting to $C$, i.e. $e(c_i, C) \in E$ where $i \in [1, m]$. There are $n$ nodes $(a_1, a_2, \ldots, a_n)$ being connected from $A$, i.e. $e(A, a_j) \in E$ where $j \in [1, n]$.

**Reduction**

*Case 1:* If there is only one node connecting to $C$, i.e. $m = 1$, then $\omega^A_C = \Delta(\omega^A_{c_1}, \omega^A_C)$.

*Case 2:* If there are more than one node connecting to $C$, i.e. $m > 1$, based on Lemma 1, $\omega^A_C = \Theta(\Delta(\omega^A_1, \omega^A_C), \Delta(\omega^A_2, \omega^A_C), \ldots, \Delta(\omega^A_m, \omega^A_C))$. It is shown that $\omega^A_C$ is solvable and unique if and only if the $\omega^A_{c_i}$ is solvable and unique where $\omega^A_{c_i}$ is the result of the sub-problem with sub-graph $G' = G - \Sigma e(c_i, C) - C$.

For Case 2, the network topology from $\{a_j\}$ to $\{c_i\}$ is unknown and maybe complex, it is possible that all $c_i$ are connected at a certain node $b$. If $b = A$, then $\hat{A}bcC$ are parallel and can be combined. If $b \neq A$, $\omega^A_{c_1}$ can be computed in as 1) the combination of $\hat{A}bcC$, or 2) combining $bcC$ first and discounting it by $\omega^A_b$. According to Lemma 1, both methods yields the same result, i.e. although we use the first method in Algorithm 1, the final result is unique.

After each reduction, $G$ is reduced into $G'$ in the way that $|E| = |E| - m$ and $|V| = |V| - 1$. Applying reductions on sub-problems recursively, finally the base case will arise, i.e. $|E| = 1$ and $|V| = 2$.

**Base Case**

The sub-graph of base case contains only one edge from $A$ to $a_j$ where $j \in [1, n]$.

Since $\omega^A_{a_j}$ is known from the original graph $G$, the base case is solvable and the result is unique. Applying the equations in Case 1 and 2 repeatedly, we can obtain an unique $\omega^A_C$ of the original problem.
Algorithm 1 ASSESS-TRUST\(G, A, C\)

Input: A directed graph \(G\) with source node \(A\) (trustor) and destination node \(C\) (trustee).

Output: \(A\)’s opinion upon \(C\), \(\omega^A_C\).

1: \(n \leftarrow 0\)
2: for each incoming edges \(e(c_i, C) \in G\) do
3: if \(c_i = A\) then
4: \(\omega_i \leftarrow \omega^A_{c_i}\)
5: else
6: \(G' \leftarrow G - e(c_i, C)\)
7: \(\omega^A_{c_i} \leftarrow \text{ASSESS-TRUST}(G', A, c_i)\)
8: \(\omega_i \leftarrow \Delta(\omega^A_{c_i}, \omega^C_{c_i})\)
9: end if
10: \(n \leftarrow n + 1\)
11: end for
12: if \(n > 1\) then
13: \(\omega^A_C = \Theta(\omega_1 \cdots \omega_n)\)
14: else
15: \(\omega^A_C = \omega_n\)
16: end if

trust” and 0 as “strongly distrust”. In addition, we add another question to let participants indicate how certain they are for their answers to the 12 questions. The uncertainty score \((Y)\) is scaled in 5 levels, where 0 represents “not sure at all” and 4 as “very confident”.

After logging into the online system [24], a participant \(A\) will first be asked to identify and evaluate the trustworthiness of her two direct friends \(B\) and \(D\). Then, \(A\) is told that her friend \(B\) trusts \(C\) with an opinion \(\omega^B_C\), and she is asked to evaluate \(B\)’s opinion on \(C\). Finally, \(A\) is told \(D\) also knows \(C\) (with an opinion \(\omega^D_C\)), and she is asked to evaluate \(C\) again, by integrating opinions from \(B\) and \(C\).

From collected data, we construct trust opinions as follows. The average score of \(X\), denoted as \(T\), reflects the portion of evidences (in a participant’s mind) that supports the target person is trustworthy. The uncertainty \(Y\) score is transferred to priori uncertainty \(e\), i.e. the more certain evidences exist, the less uncertain a participant will be in judging the target person. It’s difficult to obtain the accurate value of \(e\), since participants may not recall the exact number of evidences they used to make their judgments. However, it is noticed that people tend to form their opinions according to most recent experiences. Hence, assuming 20 recent evidences are good enough for a person to make fair judgments, the value of \(e\) is set as 1 when \(Y = 0\) and \(3/(5Y)\), otherwise. Each pair of \(T\) and \(e\) is transferred into an opinion vector according to the following equation:

\[
(b, d, n, e) = (T \cdot (1 - e), (1 - T) \cdot (1 - e), 0, e) \quad (6)
\]

Notice that we assume the posteriori uncertainty in each non-computed opinion as 0 because when human make decisions, neutral evidences are usually ignored and only those positive and negative ones take affect. In other words, we consider posterior uncertainty only occurs due to distortion when trust propagates.

B. Errors in Discounting and Combining Operations

To validate the discounting and combining operations, we calculate the errors between expected beliefs obtained from the questionnaire results and the computed ones as follows:

\[
\Delta_{Err} = \frac{E(\Delta(\omega^A_{B, C}), \omega^B_{C}) - E(\omega^A_{C})}{E(\omega^A_{B, C}) - E(\omega^A_{C})}
\]

\[
\Theta_{Err} = \frac{E(\Theta(\Delta(\omega^A_{B, C}, \omega^B_{C}), \Delta(\omega^D_{A, C})), \omega^A_{B, D}))) - E(\omega^A_{B, D})}{E(\omega^A_{C}) - E(\omega^A_{C})}
\]

where \(\omega^A_{B, C}\) denotes the opinion \(A\) hold upon \(C\) through \(B\)’s opinion, \(\omega^A_{C}\) as “very confident”.

The errors of combining and discounting operations can be seen in Fig 9. As a comparison, we also plot the results computed by subjective logic. The average error of 3VSL is less than 20%, however, the number of trust value with error less than 20% is smaller. This indicates 3VSL provides higher accuracy in multi-hop interpersonal trust assessments.

To validate the discounting and combining operations, we conduct a set of numerical analysis on the two basic operations: discounting and combining. In the numerical evaluation, we use the same parallel topology as in the previous section, and adjusts parameters \(T\) and \(e\) to see whether the model works as designed.

VI. Numerical Evaluation

To further understand the features of the 3VSL model, we conduct a set of numerical analysis on the two basic operations: discounting and combining. In the numerical evaluation, we use the same parallel topology as in the previous section, and adjusts parameters \(T\) and \(e\) to see whether the model works as designed.

A. Discounting Operation

In the serial topology shown in Fig. 1(a), we first set two opinions \(\omega^A_B\) and \(\omega^B_C\) with fixed priori uncertainty (i.e. \(e^A_B = e^B_C = 0.2\)), and vary their belief parts by changing \(T^A_B\) and \(T^B_C\). According to Eq. 6, \(\omega^B_C\) can be written as \((0.8T^B_C + 0.21 - T^A_B, 0.0, 0.2)\) and \((0.8T^B_C, 0.81 - T^B_C, 0.0, 0.2)\), respectively. As shown in Fig. 11(a), the expected belief \(E(\Delta(\omega^A_B, \omega^B_C))\) tends to approach \(T^B_C\) when \(T^A_B\) is high. When \(T^A_B\) is low, \(E(\Delta(\omega^A_B, \omega^C_B))\) approaches to 0.5. That means \(A\) tends to believe \(B\)’s opinion on \(C\) when \(A\) highly trusts \(B\); however, \(A\)’s opinion on \(C\) tends to neutral when she does not trust \(B\).
B. Combining Operation

Secondly, we evaluate the impact of both belief and priori uncertainty on combining operations. We vary $T_B^A$ and $e_B^A$ from 0 to 1, which results an opinion $(T_B^A \cdot (1 - e_B^A), (1 - T_B^A) \cdot (1 - e_B^A), 0, e_B^A)$. At the same time, we keep $\omega_C^A = (0.7, 0.1, 0, 0.2)$. As can be seen in Fig. 11(b), $E(\Delta(\omega_B^A, \omega_B^C))$ are close to $T_B^A$ when $e_B^A$ is low, and it becomes neutral when $e_B^A$ is high. This phenomena indicates when $A$ is more certain about her discretion on $B$ (i.e. smaller $e_B^A$), she will rely more on $B$ to form her opinion on $C$. Otherwise, $A$’s opinion on $C$ tends to be neutral as she cannot judge due to the lack of evidences.

C. Impact of Bridge Opinion

3VSL is able to handle the bridge topology as shown in Fig. 1(c), while subjective logic could not because it has to remove a certain edge (e.g. $\omega_B^A$). We believe the being removed edge (bridge opinion) is also important in assessing trust in the bridge topology.

By eliminating the bridge opinion (as subjective logic does), the bridge topology becomes a parallel topology where $A$ connects to $C$ by two paths $ABC$ and $ADC$. We set opinion vectors $\omega_B^A, \omega_B^A, \omega_B^C$ and $\omega_C^D$ as $(0.7, 0.1, 0, 0.2), (0.5, 0.3, 0, 0.2), (0.5, 0.3, 0, 0.2)$, and $(0.5, 0.3, 0, 0.2)$, respectively. Then, we vary the bridge opinion $\omega_B^D$ as $(1 - e_B^D)T_B^D, (1 - e_B^D)(1 - e_B^D)$. We conclude that combining two opinions with the same priori uncertainty and belief will enhance the original opinions, due to the increased evidence space. On the other hand, combing opinions with same priori uncertainty but different beliefs will neutralize these two opinions.
Fig. 13: a) Influence of bridge opinion’s belief on the resulting expected belief \( T_B^{BD} \), 0, \( e_D^B \)).

We pick a high priori uncertainty \( (e_D^B = 0.6) \) and a low priori uncertainty \( (e_D^B = 0.15) \), and vary the belief by changing \( T_B^{BD} \) from 0 to 1. As shown in Fig. 13(a), when \( e_D^B = 0.15 \), the expected belief of the bridge topology are closer to \( T_B^{BD} \) that the parallel topology. When \( e_D^B = 0.6 \), it approaches \( T_B^{BD} \) as well yet not as much as \( e_D^B = 0.15 \). It is shown that when bridge opinion is a certain one, its impact on the final result could not be omitted.

Then, we vary \( e_D^B \) from 0 to 1 and pick a high belief \( (T_B^{BD} = 0.7) \) and a low belief \( (T_B^{BD} = 0.3) \) of the opinion \( \omega_B^D \). As can be seen in Fig. 13(b), when \( e_D^B \) is low, the impact of the bridge opinion should not be ignored. When \( e_D^B \) is high, the expected belief gets close to the parallel topology. That means when \( e_D^B \) is high (no enough evidences), the bridge opinion can be ignored as what the subjective logic does. However, when the \( e_D^B \) is low, it has significant impact on the final result and must be taken into account. Moreover, we note that a higher belief of the bridge opinion leads to a larger expected belief than the parallel topology and verse versa.

VII. CONCLUSION AND FUTURE WORK

Three-valued subjective logic is proposed to compute the interpersonal trust between any two persons who have not had interactions before. 3VSL introduces posteriori uncertainty space to store the evidences distored from certain spaces as trust propagates, and priori uncertainty space to control the evidence size as trusts combine. We also discover the differences between distorting and original opinions, i.e. original opinions are so unique that they can be reused in trust computation while distorting opinions are not. We validate 3VSL both in theory and real world evaluation. The results indicate that 3VSL is sound and can be applied in computing trust with high accuracy. For the future work, we will improve 3VSL by employing stochastic process to model a trust opinion. We will also evaluate 3VSL in real-word scenarios with more complicated topologies. In addition, Bayesian analysis will be integrated to make 3VSL be able to handle evidences from multiple sources.

REFERENCES


